MASS MEASUREMENTS IN NUCLEAR EMULSIONS BY MULTIPLE SCATTERING AND GAP DISTRIBUTION

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ABSTRACT. A method of identifying singly charged particles in nuclear emulsions with the help of gap distribution and multiple coulomb scattering measurements for the same track is described. A parameter S depending upon scattering parameter $\alpha_{100\mu}$ and gap distribution parameter g^* is obtained, which is a logarithmic function of the mass. The observed values of S are in good agreement with the calculated ones.

INTRODUCTION

Particle identification in nuclear emulsions, in the case of singly charged particles reduces simply to the determination of the mass of the particles producing the given tracks. Accurate determination of mass has assumed new importance in recent years because of the reported existence of a large number of new particles. The nature of many of these new particles is not very well understood.

In order to find out the mass of an unknown particle producing a track in photographic emulsion, two parameters are necessary, which are functions of velocity and mass. One of them is generally taken to be the ionization produced by the particle. Information of the ionization produced can be had from the measurements of the grain density, blob density, photometric track density, and by studying gap-length distribution.

The other parameter is either the residual range or the mean square angular deviation suffered by the particle as a result of multiple scattering in the emulsion. The last two by themselves are also sufficient to specify the mass.

In a recent paper, Fowler and Perkins (1955), have shown that the distribution of gap-lengths occuring in tracks of ionizing particles is exponential over the entire range of ionization measured and that the coefficient g of this exponential is the most useful measure of the ionization in the track. The normalised value of g is $g^* = g/g_0$ where g_0 corresponds to minimum ionization. g^* is independent of the degree of development of the emulsion. Moreover the value of of g^* does not depend on the developed grain size. This method of determination of ionization is as accurate as the photometric method and has the additional advantage of consuming less time.

Multiple coulomb scattering gives a measure of the produ t of momentum and velocity, i.e. $p\beta$ of the particle. It has been thoroughly investigated both theoretically and experimentally. Recent advances on the "Constant Cell Method" of Fowler (1950) have culminated in the "Constant Sagitta Methcd" described by Biswas *et al* (1953) and Dilworth *et al* (1954). The constant cell method after the elimination of various kinds of noise, which have been discussed by Menon *et al* (1951) and Biswas *et al* (1955), gives accurate results, (specially for fast particles.

In the method developed in this paper the scattering parameter $\bar{\alpha}_{100\mu}$ and the gap distribution parameter g^* are determined for the same tiack. We h ve obtained a parameter S which depends upon $\bar{\alpha}_{100\mu}$ and g^* and is a logarithmic function of the mass of the particle. This relation enables us to determine the mass with reasonable accuracy. The values of S calculated according to our relation are in good agreement with those obtained from values of $\bar{\alpha}_{100\mu}$ and g^* given by Glasser (1955) and Fowler and Perkins (1955) respectively.

II. DETERMINATION OF THE RELATIONSHIP BETWEEN $\bar{\alpha}_{100\mu}$ AND g^*

Scattering Parameter $\bar{\alpha}_{100\mu}$:—According to Voyvodic and Pickup (1952), for Ilford G5 emulsions, the mean absolute deflection suffered by a particle undergoing multiple coulomb scattering is given by

$$\bar{\alpha} = Kt^{\frac{1}{2}}/pv \quad \text{degrees} \qquad \dots \ (1)$$

p, v are the momentum and velocity respectively of the charged particle; t is the cell length in microns and K is referred to as the "scattering constant" in degrees Mev/$(100\mu)^{\frac{1}{2}}$. K for Ilford G5 emulsion varies from 24 to 29 and is generally taken as 26 Mev degrees/$(100\mu)^{\frac{1}{2}}$ ... (2)

In emulsion work, it has been a useful convention to measure the quantity $\bar{\alpha}_{100\mu}$ which is defined as [Menon and O'Ceallaigh (1953)]

$$\bar{\alpha}_{100\mu} = \frac{1}{B_1 - B_2} \int_{B_2}^{B_1} \bar{\alpha} t^{-\frac{1}{2}} dB \qquad \dots \ (3)$$

where $B = (1-\beta^2)^{-\frac{1}{2}}$ and B_1, B_2 are the values of B at the two ends of the section of a track along which multiple scattering is measured.

Using (1) and (3) we get

$$\bar{\alpha}_{100\mu} = \frac{1}{B_1 - B_2} \int_{B_2}^{B_1} K/pv \, dB \qquad \dots \ (4)$$

Gap-distribution Parameter g^*:—As mentioned earlier, Fowler and Perkins (1955) have shown that in the tracks produced by ionizing particles, the gap length distribution is exponential over the entire range of ionization. The coefficient g of this exponential is a useful measure of ionization and is given by

$$g = \frac{1}{l_2-l_1} \ln \frac{H_1}{H_2} \quad \dots \quad (5)$$

where H_1 is the number of gaps exceeding length l_1 and H_2 is the number of gaps exceeding length l_2. Moreover for values of ionization less than ten times the minimum ionization

$$g \propto \frac{dE}{dR} \quad \dots \quad (6)$$

dE/dR gives the rate of energy loss and is a function of velocity only.

The normalized value of g i.e. g^* is related to the residual range R measured in microns in the case of protons as

$$g_P^* = aR_P^m \quad \dots \quad (7)$$

where

$$m = -0.42 \quad \dots \quad (8)$$

$$a = 2.93 \times 10^2 \quad \dots \quad (9)$$

The constant of proportionality a in (7) is determined from the g^*-R curve for protons given by Fowler and Perkins (1955).

Range-energy Relation:—The accepted range-energy relation for Ilford G5 emulsions in the case of protons is [Glasser (1955)]

$$R_P = 10.6\, E_P^{1.68}$$

or

$$E_P = kR_P^n \quad \dots \quad (10)$$

where

$$k = 0.246 \quad \dots \quad (11)$$

$$n = 0.595 \quad \dots \quad (12)$$

The general range-energy relation for the case of any singly charged particle can be written as

$$E = kM^{1-n}\, R^n \quad (13)$$

where M is the mass of the particle in proton units.

Determination of S_p: Let us choose a function of $\bar{\alpha}_{100\mu}$ and g^* which will be easily amenable to the theoretical and experimental treatment. After a large number of trials, we arrived at the following function

$$S = \log \bar{\alpha}_{100\mu} - 1.5 \log g^*$$

This function was subsequently slightly modified and we now define S as

$$S = \log \bar{\alpha}_{100\mu} + \frac{n}{m} \log g^* \qquad \dots \quad (14)$$

where 'm' and 'n' are given by (8) and (12) respectively.
We can proceed to determine the value of S_p (the value of S for protons) as follows:

Let us rewrite (4) after a little simplification as

$$\bar{\alpha}_{100\mu} = \frac{K}{2\mu(B_1 - B_2)} \log \frac{1-B_1}{1-B_2} + \frac{K}{2\mu + (1+B_2)}$$

μ is the mass of the particle in Mev.

The second term on the right is a small correction term and can be neglected, so that

$$\bar{\alpha}_{100\mu} = \frac{K}{2\mu(B_1 - B_2)} \log \frac{1-B_1}{1-B_2}$$

If we put $\bar{B} = (B_1+B_2)/2$ and $b = B_1 - B_2$. then

$$\bar{\alpha}_{100\mu} = \frac{K}{2\mu}\frac{1}{b} \log \left[\left(1 - \frac{b/2}{1-\bar{B}}\right) \Big/ \left(1 + \frac{b/2}{1-\bar{B}}\right)\right]$$

$$\approx \frac{K}{2\mu}\frac{1}{\bar{B}-1} \qquad \dots \quad (15)$$

The range-energy relation (10) can also be expressed as

$$\mu_P[B-1] = k R_P^n$$

or $$B - 1 = k \mu_P^{-1} R_P^n$$

Thus $$\bar{B} - 1 = k\mu_P^{-1} (R_{P1}^n + R_{P2}^n)/2$$

Let the scattering measurements be made on the faster half of the track so that B_1 and B_2 represent velocities corresponding to ranges R_p and $R_p/2$ respectively So we have

$$\bar{B} - 1 = k\mu_P^{-1} \frac{2^n+1}{2.2^n} R_P^n \qquad \dots \quad (16)$$

Eliminating $(\bar{B}-1)$ and R_P from (7), (10), and (15) one gets

$$(\bar{\alpha}_{100\mu})_P = \frac{K}{2K'} g_P^{*\,-n/m}$$

or $$\log(\bar{\alpha}_{100\mu})_P + \frac{n}{m}\log g_P^* = \log\frac{K}{2K'} = \text{constant} \qquad \dots (17)$$

where $$K' = k\,\frac{2^n+1}{2.2^n}\,a^{-n/m} \qquad \dots (18)$$

From (2), (8), (9), (11), (12), and (18), we have

$$\log\frac{K}{2K'} = -1{\cdot}70$$

Hence $$S_P = \log(\bar{\alpha}_{100\mu})_P + \frac{n}{m}\log g_P^* = -1{\cdot}70 \qquad \dots (19)$$

Determination of S: — We can find the value of S for any singly charged particle as follows:

If two particles have the same velocity, then their ranges are proportional to their masses. Thus if μ, μ_P represent masses of a singly charged particle and of a proton respectively, in energy units, and R and R_P be the respective ranges, then

$$R_P = (\mu_P/\mu)R \qquad \dots (20)$$

Also it follows from (5) and (6) that the value of g and hence of g^* is the same for two particles of the same charge and velocity having different masses. Hence for any singly charged particle we get

$$g^* = a(\mu/\mu_P)^{-m}\,R^m$$
$$= a'\,\mu^{-m}\,R^m \qquad (21)$$

Taking $\mu_P = 931$ Mev., $a' = a(931)^m$ (22)

Also general range-energy relation (13) can be written as

$$\mu[B-1] = k(\mu/\mu_P)^{1-n}\,R^n$$
$$B-1 = k'\mu^{-n}\,R^n \qquad (23)$$

Here $$k' = k(931)^{n-1} \qquad (24)$$

As before from (23) $\bar{B}-1 = k'\,\mu^{-n}\,\frac{2^n+1}{2.2^n}\,R^n$ (25)

Eliminating R and $(\bar{B}-1)$ from (15), (21), and (25), we have

$$\bar{\alpha}_{100\mu} = \frac{K}{2K''}\,\mu^{-1}g^{*-n/m} \qquad \text{where } K'' = k'\,\frac{2^n+1}{2.2^n}\,a'^{-n/m}$$

or $$\log\bar{\alpha}_{100\mu} + \frac{n}{m}\log g^* = \log\frac{K}{2K''} - \log\mu$$

Hence $$S = A - \log\mu \qquad \dots (26)$$

$$\text{constant } A = \log \frac{K}{2K''}$$

This parameter S depends upon $\bar{\alpha}_{100\mu}$ and g^* by definition. Knowing the values of these two quantities for a given track, we can determine this parameter and hence the mass of the unknown particle from (26). Thus the mass estimate depends upon the experimental determination of the statistical parameter S which is a logarithmic function of the mass. We are to keep in mind that $\bar{\alpha}_{100\mu}$ is determined for the faster half of the track.

Substituting the values of various constants a', k', n, m, and K, we obtain $A = 1.36$, so that (26) becomes

$$S = 1.36 - \log \mu \qquad \ldots \quad (26a)$$

If we substitute for μ for various singly charged particles, we can find the corresponding values for S. We can, thus, have a scale of S-values

$$S_\mu,\ S_\pi,\ S_K,\ S_P,\ S_D,\ S_T \ldots\ldots$$

for μ-meson, π-meson, K-meson, proton, deutron, triton etc. Knowing these values and finding the S-value for the unknown particle, we can at once identify it.

III. EXPERIMENTAL VERIFICATION

In order to test the correctness of relations (19) and (26), let us find out the values of S_P and S_π experimentally. For the tracks considered, the values of $\ddot{\alpha}_{100\mu}$ and g^* for protons and π-mesons are to be determined.

In the following table the values of $\bar{\alpha}_{100\mu}$ have been taken from the paper by Glasser (1955) wherein an experimental relationship has been found between scattering and range

$$<\bar{\eta}> t = (19{\cdot}0 \pm 0.3)(M_P/M)^{0.393 \pm 0.016} \times (t/50)^{3/2} R^{-(0.607> \pm 0.016)} \quad \ldots \quad (27)$$

t is the cell length in microns and M_P, M are masses of proton and any other singly charged particle respectively.

If y_i is the projection of the track on the axis at right angles to the one to which the track is made parallel and y_i is measured after constant cell length t microns, then

$$<\eta_i> t = y_i - \tfrac{1}{2}(y_{i-1} + y_{i+1}) \text{ microns}$$

Usually, we determine the second differences of the projections

i.e.
$$<D_i> t = 2y_i - (y_{i-1} + y_{i+1}) \text{ microns}$$
$$= 2 <\eta_i> t$$

So we have $$\bar{\alpha}_t = \frac{<\bar{D}>t}{t} \times \frac{180}{\pi} \text{ degrees}$$

and hence $$\bar{\alpha}_{100\mu} = \frac{2<\bar{\eta}>_t}{t} \times \frac{180}{\pi} \times \left(\frac{100}{t}\right)^{\frac{1}{2}} \text{ degrees}$$

$$\text{or } \bar{\alpha}_{100\mu} = 3.24<\bar{\eta}>_{50\mu} \qquad \dots \quad (28)$$

$\bar{\alpha}_{100\mu}$ is then determined from (27) and (28) for various values of R.

The values of $g_p{}^*$ have been determined from the paper by Fowler and Perkins (1955) who have given a curve showing the relationship between g_p^* and R_p. The values of $g_\pi{}^*$ have been determined from the corresponding values of g_p^* for the same residual ranges by use of (29) obtained from (7) and (21).

$$g_\pi{}^* = g_p{}^*(\mu_\pi/\mu_p)^m \qquad \dots \quad (29)$$

The values of S_p and S_π are then determined from (14).

R	g_p^*	g_π^*	$(\alpha_{100\mu})_P$	$(\alpha_{100})_\pi$	S_p	S_π
0.1 cm	14.0	6.29	0.931	1.97	−1.66	−0.84
0.5 cm	8.6	3.86	0.351	0.74	−1.78	−0.96
1.0 cm	6.2	2.78	0.230	0.49	−1.76	−0.94
4.0 cm	3.5	1.57	0.099	0.21	−1.77	−0.96
10.0 cm	2.25	1.01	0.057	0.12	−1.74	−0.93

Mean $S_p = -1.74$ and $S_\pi = -0.93$

The value of S_p from (19) is -1.70 and that of S_π from (26′) is -0.88, which are in good agreement with the values obtained above.

IV CONCLUSION

Let us examine (26) critically. First of all we have assumed that A is a constant which amounts to saying that scattering constant K has a fixed value. This, however, is not so, since K depends upon the velocity of the particle and the cell-length employed. Although this dependence is slight and may be neglected in ordinary work, this must be taken into account in precision mass measurements of unknown particles. A correction for the variation in the value of K can be made as done by Menon and O'Ceallaigh (1953).

Secondly, $\bar{\alpha}_{100\mu}$ is measured along the faster half of the track. In general, if B', B'' measure velocities at residual ranges R', R'' respectively, then instead of $\bar{B}$, we should use the value of B_{eff} which corresponds to residual range R_{eff}, which is given by the following relation due to Menon and Rochat (1951)

$$R_{eff} = \left[\frac{1}{R''-R'} \int_{R'}^{R''} \frac{dR}{R^{2n}} \right]^{-\frac{1}{2}n}$$

where 'n' is given by (12).

Statistical errors in the evaluation of g^* have been discussed by Fowler and Perkins (1955) and the various errors in the measurement of $\bar{\alpha}_{100\mu}$ by Menon *et al* (1951) and Biswas *et al* (1955). Taking an extreme case, if $\bar{\alpha}_{100\mu}$ is measured with an error of 10% and g^* with an error of 5%, then the percentage error in S will be $\approx 12/S$ which in the case of protons is $\approx 7\%$.

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